Mathematical League of University of Lodz

Series II 24/25

For every exercise you can get max. 10. p. Solutions should be delivered on paper (every task on the separate piece of paper) to the room B207 or electronically on the address: piotr.nowakowski@wmii.uni.lodz.pl. Deadline: 17.01.25.

Exercise 1. *Let* $n \in \mathbb{N}$ *and* $x_1, x_2, \ldots, x_n > 0$ *be such that*

$$\frac{1}{1+x_1} + \frac{1}{1+x_2} + \ldots + \frac{1}{1+x_n} = 1.$$

Show that

$$\sqrt{x_1} + \sqrt{x_2} + \ldots + \sqrt{x_n} \geq (n-1)\left(\frac{1}{\sqrt{x_1}} + \frac{1}{\sqrt{x_2}} + \ldots + \frac{1}{\sqrt{x_n}}\right).$$

Exercise 2. *Let* $s_1, s_2, \ldots, s_n$ *be switches connected to one light* S. *Assume that every switch can be in one of three states, and also the light* S *can be in one of three states. The state of the light* S *depends on the configurations of stages of switches* $s_1, \ldots, s_n$. *Moreover, we know that if we change the states of all switches, then the state of* S *also changes. Prove that there exists a switch* s_i *with the property that the state of the light* S *depends only on the state of* s_i.

Exercise 3. *Let* A, B, C, D, E, F, G, H *be consecutive vertices of the octagon inscribed in a circle. It is known that* A, C, E, G *is a square with area* 5 *and* B, D, F, H *is a rectangle with area* 4. *What is the maximum area of the octagon?*